AVIRAL CLASSES
IIT-JEE | NEET | FOUNDATIONS

MATHEMATICS
CRASH COURSE

LECTURE-03

TOPICS : Sequence and Series

1. The sum to n terms of the series

$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+.....$ is

(a) $\frac{3n}{n+1}$

(b) $\frac{6n}{n+1}$

(c) $\frac{9n}{n+1}$

(d) $\frac{12n}{n+1}$

2. For two unequal numbers, A.M. = 4 and G.M. = 2. Then H.M. =

(a) 1/2 (b) 1

(c) 2 (d) 3

3. If the m^{th} term of a H.P. be n and the n^{th} term be m, then the $(m\ n)^{th}$ term is

(a) 1/2 (b) 1/3

(c) 1 (d) 2

4. If $\log_2(5.2^x + 1)$, $\log_4(2^{1-x} + 1)$ and 1 are in A.P., then x equals

(a) $\log_5 2$

(b) $1 - \log_2 5$

(c) $\log_2 5$

(d) None of these

5. If $a_1, a_2, a_3, ...$ is an A.P. such that

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ then

$a_1 + a_2 + a_3.... + a_{24} =$

(a) 75 (b) 750

(c) 900 (d) 90

6. $x^{1/2}.\ x^{1/4}.x^{1/8}..\ x^{1/16}$ ... to ∞ is equal to

(a) 0 (b) 1

(c) x (d) None of these

7. Let S_n denote the sum of first n terms of an A.P. if $S_{2n} = 3\ S_n$, then the ratio S_{3n}/S_n is equal to

(a) 4 (b) 6

(c) 8 (d) 10

8. If $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P., then a, $\frac{1}{b}$, c are in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

9. If the ratio between the sums of n terms of two A.P.'s is 3n + 8 : 7n + 15, then the ratio between their 12^{th} terms is

(a) 7 : 16

(b) 16 : 7

(c) 74 : 169

(d) 169 : 74

10. If x > 1, y > 1, z > 1, are in G.P., then

$\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in

(a) A.P.

(b) G.P.

(c) H.P.

(d) None of these

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TOPICS : Sequence and Series SOLUTION

1	B
2	B
3	C
4	B
5	C
6	C
7	B
8	C
9	A
10	C

1. (b) Let T_r be the rth term of the given series. Then,

$$T_r = \frac{2r+1}{1^2+2^2+.....+r^2}$$

$$= \frac{2r+1}{(r/6)(r+1)\,(2r+1)} = 6\left(\frac{1}{r}-\frac{1}{r+1}\right)$$

So, required sum $= \sum_{r=1}^{n} T_r = 6\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$

$$= 6\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$= 6\left[1-\frac{1}{n+1}\right] = \frac{6n}{n+1}$$

2. (b) Since A.M., G.M. and H.M. are in G.P., therefore

$$G^2 = AH \Rightarrow 4 = 4H \Rightarrow H = 1$$

3. (c) Let a be the first term and d be the common difference of the corresponding $A.P.$ Then,

$$n = m^{th} \text{ term} = \frac{1}{a+(m-1)d} \Rightarrow a + (m-1)\,d = \frac{1}{n} \quad \ldots (i)$$

Similarly, $a + (n-1)\,d = \dfrac{1}{m}$ ... (ii)

Solving these two equations, we get $a = \dfrac{1}{mn}$, $d = \dfrac{1}{mn}$.

So $(mn)^{th}$ term of the H.P.

$$= \frac{1}{(mn)^{th} \text{ term of the corresponding A.P.}}$$

$$= \frac{1}{a+(mn-1)d} = \frac{1}{\frac{1}{mn}+(mn-1)\frac{1}{mn}} = 1$$

4. (b) $\because$ The given numbers are in $A.P.$

$\therefore$ $2\log_4(2^{1-x}+1) = \log_2(5.2^x+1)+1$

$\Rightarrow$ $2\log_{2^2}\left(\dfrac{2}{2^x}+1\right) = \log_2(5.2^x+1)+\log_2 2$

$\Rightarrow$ $\dfrac{2}{2}\log_2\left(\dfrac{2}{2^x}+1\right) = \log_2(5.2^x+1)\,2$

$\Rightarrow$ $\log_2\left(\dfrac{2}{2^x}+1\right) = \log_2(10.2^x+2)$

$\Rightarrow$ $\dfrac{2}{2^x}+1 = 10.2^x+2$

$\Rightarrow$ $\dfrac{2}{y}+1 = 10y+2$, where $2^x = y$

$\Rightarrow$ $10y^2+y-2=0 \Rightarrow (5y-2)(2y+1)=0$

$\Rightarrow$ $y = 2/5$ or $y = -1/2$. $\Rightarrow$ $2^x = 2/5$ or $2^x = -1/2$

$\Rightarrow$ $x = \log_2(2/5)$ [$\because$ $2x$ can not be negative]

$\Rightarrow$ $x = \log_2 2 - \log_2 5$

$\Rightarrow$ $x = 1 - \log_2 5$

5. (c) we have

$$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$$

$\Rightarrow$ $(a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow$ $3(a_1 + a_{24}) = 225$

$\because$ In A.P., nth term from beginning + nth term from end = 1st term + last term

$\Rightarrow$ $a_1 + a_{24} = 75$ ... (1)

$\therefore$ $a_1 + a_2 + a_3 + a_4 + \ldots\ldots\ldots\ldots + a_{24}$

$\Rightarrow$ $(a_1 + a_{24}) + (a_2 + a_{23}) + \ldots\ldots\ldots\ldots + (a_{12} + a_{13})$

$12(a_1 + a_{24}) = 12.75 = 900$

6. (c) We have,

$$x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} \cdot x^{\frac{1}{16}} \ldots \text{to } \infty$$

$$= x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots \text{ to } \infty} = x^{\frac{1/2}{1-1/2}} = x$$

7. (b) Given that $S_{2n} = 3S_n$

$\Rightarrow$ $\frac{2n}{2}[2a + (2n-1)d] = \frac{3n}{2}[2a + (n-1)d]$

$\Rightarrow$ $2a = (n+1)d.$

Now $\frac{S_{3n}}{S_{2n}} = \frac{3n/2[2a + (3n-1)d]}{n/2[2a + (n-1)d]}$

$$= \frac{3[(n+1)d + (3n-1)d]}{[(n+1)d + (n-1)d]} = 6$$

8. (c). $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P.

$\Rightarrow$ $b - \frac{a+b}{1-ab} = \frac{b+c}{1-bc} - b$

$\Rightarrow$ $\frac{a(b^2+1)}{1-ab} = \frac{c(b^2+1)}{1-bc}$ $\Rightarrow$ $-\left(\frac{1-ab}{a}\right) = \frac{1-bc}{c}$

$\Rightarrow$ $-\frac{1}{a} + b = \frac{1}{c} - b$ $\Rightarrow$ $2b = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow$ $a, \frac{1}{b}, c$ are in H.P.

9. (a) Replacing n by $(2 \times 12 - 1)$ *i.e.* 23, we have :

$\therefore$ Required ratio $= \dfrac{3 \times 23 + 8}{7 \times 23 + 15} = \dfrac{7}{16}$

10. **(c) Since x, y, z are in G.P. Therefore $y^2 = xz$**

$\Rightarrow \quad 2 \ln y = \ln x + \ln z$

$\Rightarrow \quad 2 + 2 \ln y = (1 + \ln x) + (1 + \ln z)$

$\Rightarrow \quad 2(1 + \ln y) = (1 + \ln x) + (1 + \ln z)$

$\Rightarrow \quad 1 + \ln x,\ 1 + \ln y,\ 1 + \ln z$ are in A.P.

$\Rightarrow \quad \dfrac{1}{1 + \ln x}, \dfrac{1}{1 + \ln y}, \dfrac{1}{1 + \ln z}$, are in H.P.